

# *Enhancing earth system decision making with Dominant Frequency State Analysis*

John T. Bruun<sup>1,2</sup> and Katy L Sheen<sup>1</sup>

1: University of Exeter, Geography, Pilot project (M2DPP035)

[k.l.sheen@exeter.ac.uk](mailto:k.l.sheen@exeter.ac.uk), [j.brunn@exeter.ac.uk](mailto:j.brunn@exeter.ac.uk)

2: Independent Academic (CPhys MInstP) [johnbruunphysics@gmail.com](mailto:johnbruunphysics@gmail.com)

[IOP Physics Communicators Group](#), [IOP Women in Physics Group](#), [@johnsoundspace](#)

Decision Making Under Uncertainty: M2D Conference 2018, Isaac Newton Institute, Cambridge

Key themes: Food security, physical processes, reducing uncertainty with both sophistication & simplicity, climate physics, Sahel, universality, extreme value processes, dominant frequency state analysis (DFSA)

*Red box: some open discussion, M2D extension, collaboration suggestions....*

**John:** Theoretical Physicist (Lancaster, 1994) and communicator

**A. ENSO dynamics, mesoscale transport, DFSA**

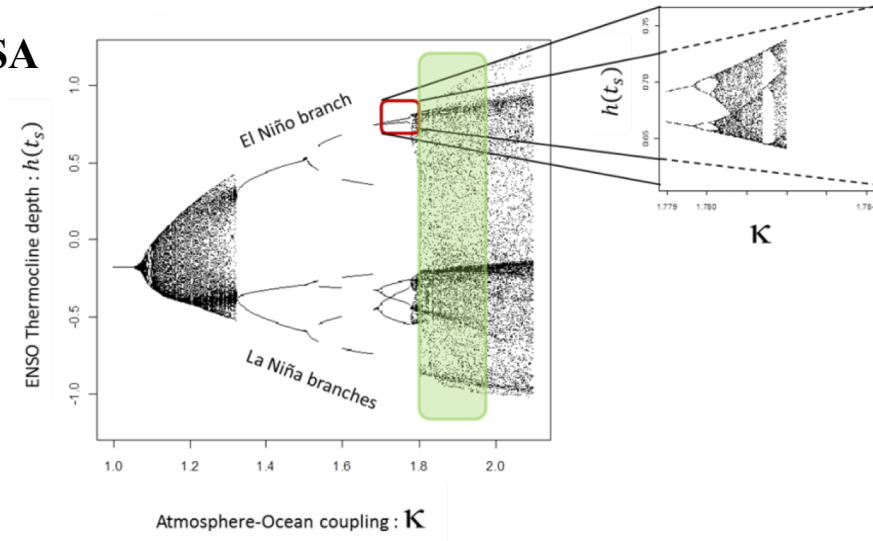
Bruun, *et al.* (2017), Heartbeat of the Southern Oscillation explains ENSO climatic resonances, *J. Geophys. Res. Oceans*, 122, 6746.

**B. Universality (Physical, Statistical)**

Inc. dynamics, Extreme value theory and cascade risks

**C. System identification approaches**

**D. Global and local social-economic and policy analysis**



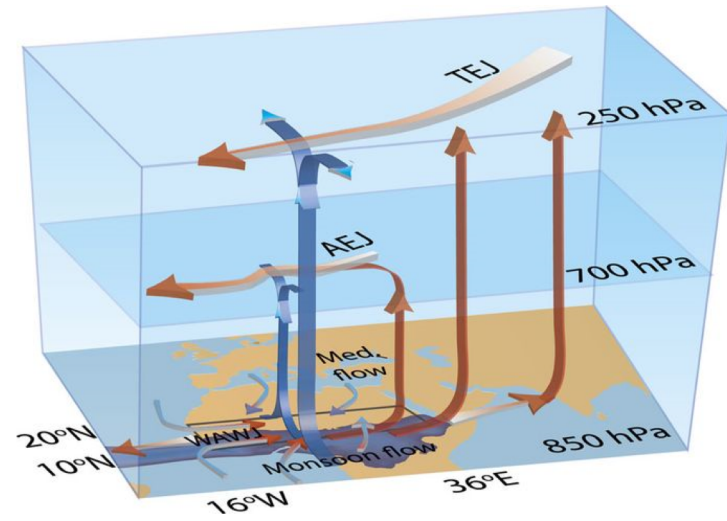
**Katy:** Physical Geographer (Cambridge, 2010) and communicator

**A. Climatic fluid flow dynamics in Southern Ocean**

**B. Sahel rainfall dynamics and model skill**

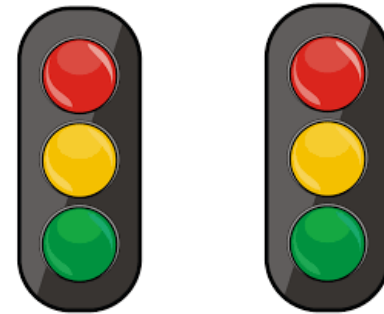
Sheen *et al.* (2017), Skilful prediction of Sahel summer Rainfall on inter-annual and multi-year time scales *Nature Communications*, 8, 14966.

**C. Global and regional Aerosol pollution studies**

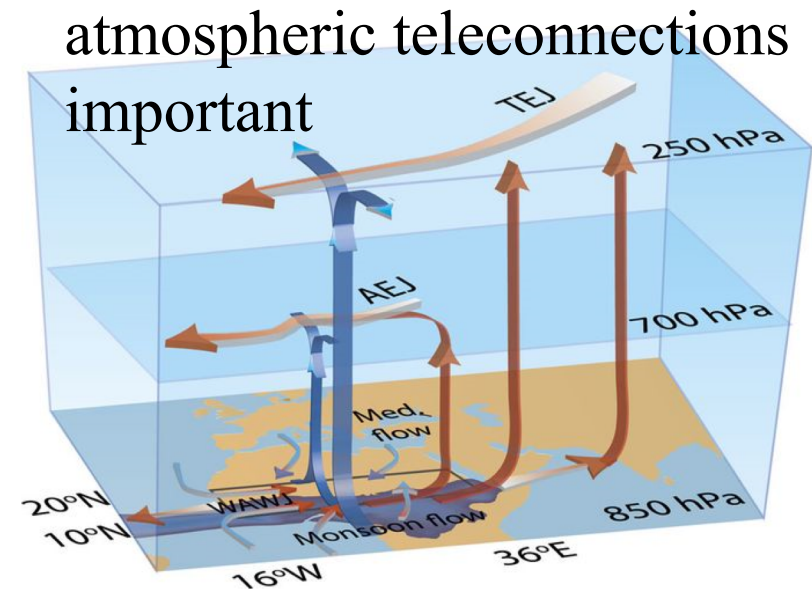


The Sahel rain budget is influenced by planetary scale climatic phenomena

Resilience/food security linked to rain pattern extremes: better understanding enhances our decision making and reduces community risk  
Using 'traffic light' rain budget risk maps

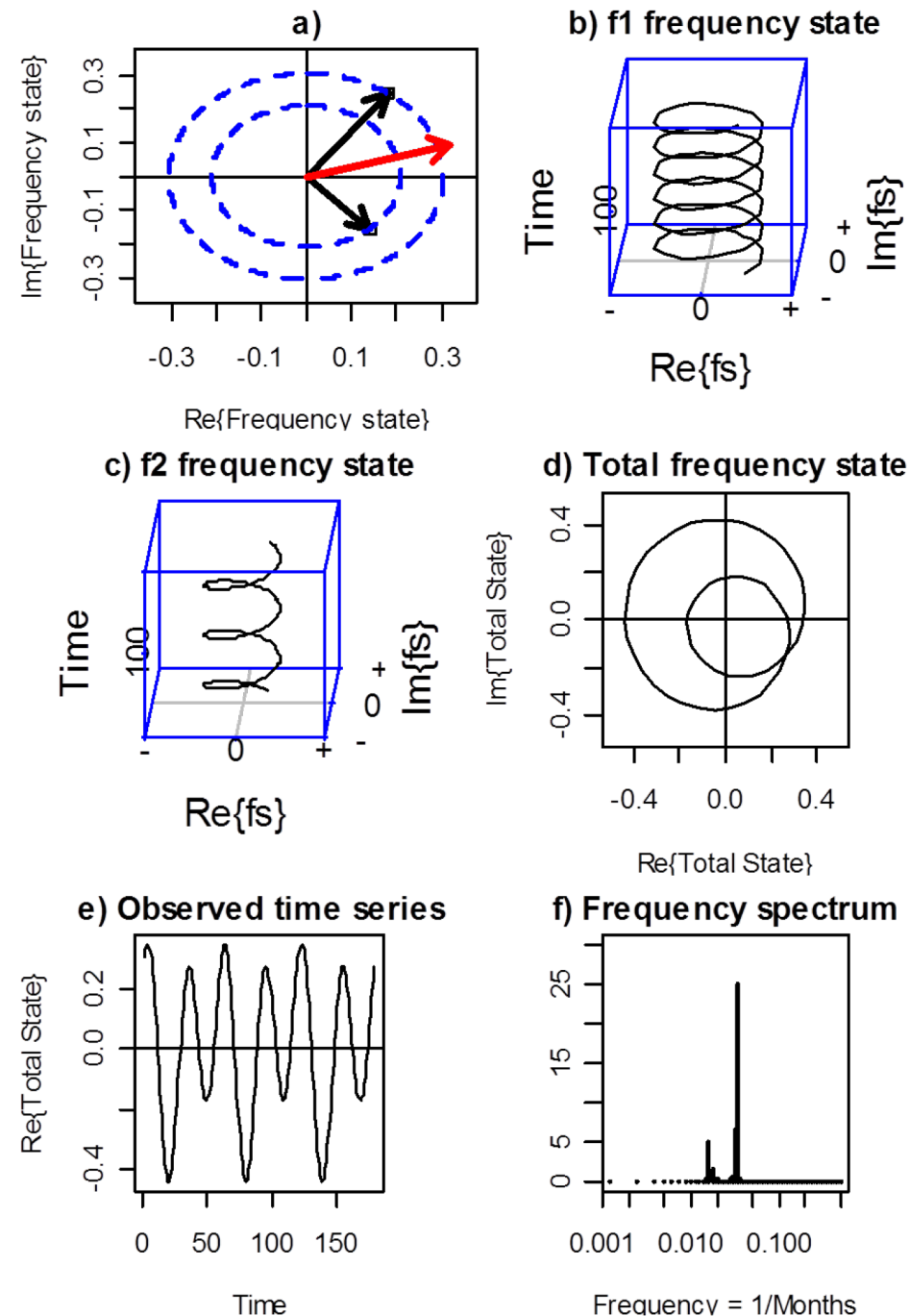


The dynamics are complex...  
ENSO impacts arid conditions



# DFSA: motivation non-linear resonance in a periodic dynamic oscillator systems.

- Eigenvectors  $f_1$ ,  $f_2$  and total combination (scaled red arrow)
- $f_1$  frequency state
- $f_2$  frequency state (1/2 'speed')
- total frequency state (period three orbit)
- observed time series
- frequency spectrum of observed time series.



Apply DFSA to determine Earth System modes for any variable. Dynamic characteristics and coloured noise



# Example: Identify and estimate non-linear universal properties of non-linear Periodic oscillator

Using the logistic map (period doubling characteristic)

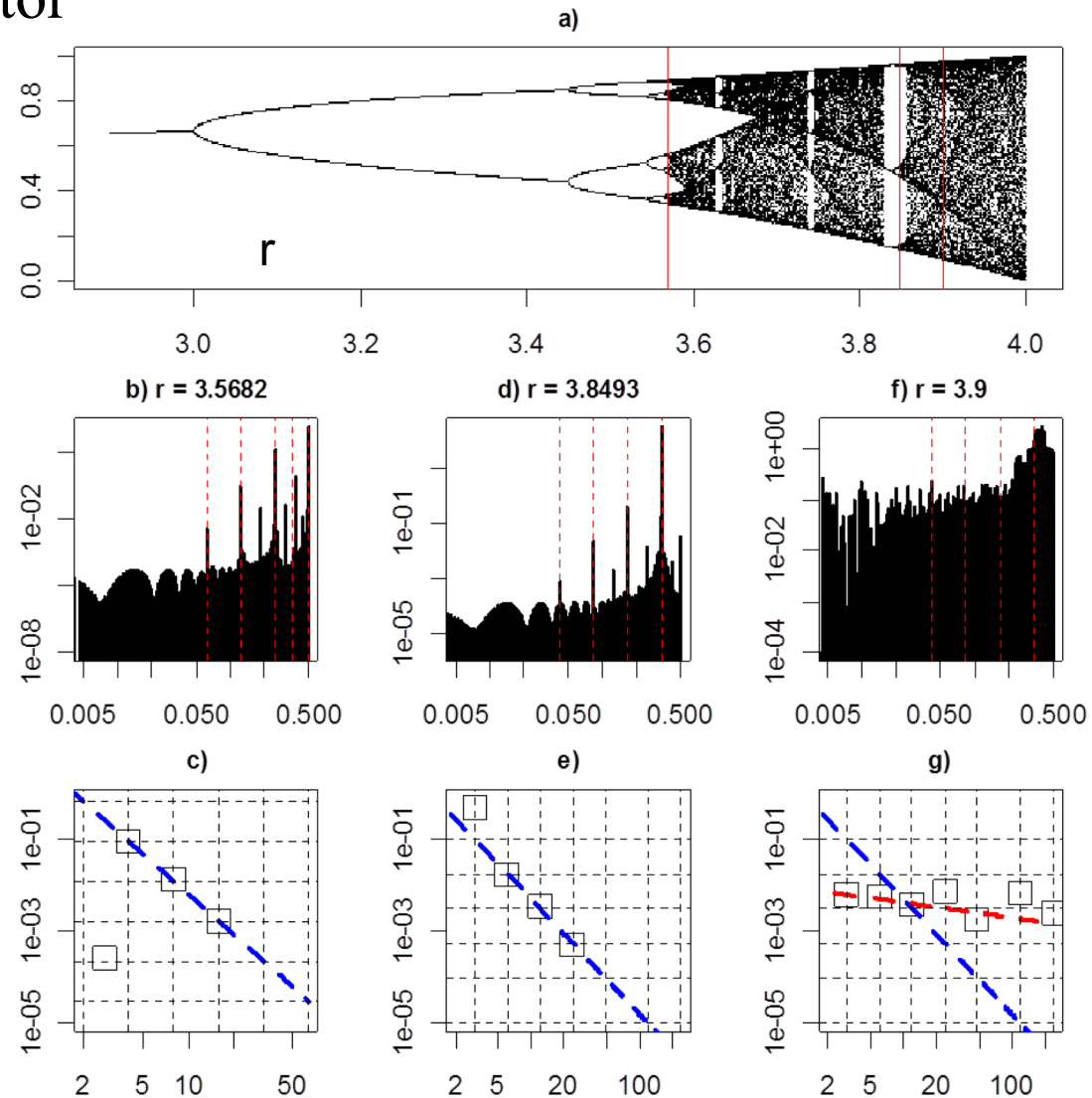
The eigenvalue spectrum has a universal subharmonic decay:

$$2n^2 : 8.7dB$$

$$3n^2 : 7.6dB$$

$$3n^2 + Intermittency : 1dB$$

(Feigenbaum, 1980;  
Bruun et al., 1994, 2017)



# Dominant Frequency State Analysis (Basic method)

*Eigenvalue equation  
(a-priori structure on rhs)*

$$D \{\psi(\mathbf{r}, t)\} = \lambda \psi(\mathbf{r}, t) \quad (1)$$

*Frequency state decomposition*

$$\lambda \psi(\mathbf{r}, t) = \sum_{k=1}^n \underline{f}_k^s(\mathbf{r}, t) = \underline{f}^s(\mathbf{r}, t) \quad (2)$$

$$\underline{f}_k^s(t) = A_k e^{i(\omega_k t + \phi_k)} = A_k \{\cos(\omega_k t + \phi_k) + i \sin(\omega_k t + \phi_k)\} \quad (3)$$

*Observation of frequency states*

$$y(t) = \sum_{k=1}^n \text{Re} \{ \underline{f}_k^s(\mathbf{r}_o, t) \} + \alpha I(\mathbf{r}_o, t) + \eta(\mathbf{r}_o, t) \quad (4)$$

The  $\omega_{process} = \{\omega_1, \omega_2, \dots, \omega_n\}$  are the a-priori identified dominant frequencies at relevant spatial location  $\mathbf{r} = \mathbf{r}_o$ ,  $\alpha I(\mathbf{r}_o, t)$  are intervention terms and  $\eta$  is a noise process.

*Example application to ENSO including thresholds*

$$\lambda \psi(\mathbf{r}_o, t) = ENSO(t) : \begin{cases} \mathbf{r}_o = \text{tropical Pacific} \\ \mathbf{r}_o = \text{tropical and extratropical Pacific} \\ \mathbf{r}_o = \text{Pacific basin} \end{cases} \quad (5)$$

$$ENSO(t) = \sum_{k=1}^n f_k^s(t) + \eta(t) \quad \text{for } A_- < \sum_{k=1}^n f_k^s(t) < A_+ \quad (6)$$

with  $f^s = \text{Re} \{ \underline{f}^s \}$ , time  $t$ ,  $n$  distinct frequency states and random noise (and red-noise)  $\eta(t)$

*Estimation with likelihood  $l(\theta)$  inference (no other estimators can asymptotically be more efficient)*

$$\nabla_{\theta} \hat{l}(\hat{\theta} \mid \text{observed data}) = 0 \quad (7)$$

# Extending DFSA to resolve spatio-temporal structure

EOF's typically separate the spatial and time components:

$$\lambda \psi(\mathbf{r}, t) \rightarrow u(\mathbf{r})v(t)$$

With DFSA all the spatial dynamic information is mapped onto the time axis, and an assumption of separation is not required.

**Map out the dynamic structure of planetary phenomena  $\lambda \psi(\mathbf{r}, t)$**

*Use a rapid analysis approach  $\lambda \psi(r_{ijk}, t) \forall i, j, k \rightarrow v_{ijk}(t)$*

*Assess heterogeneity through similarity of  $v_{ijk}(t)$ ;  $\forall i, j, k$  to give  $B_l$  sets.*

Sahel downscaling rain budget maps:  $Z(\mathbf{r}, t)$

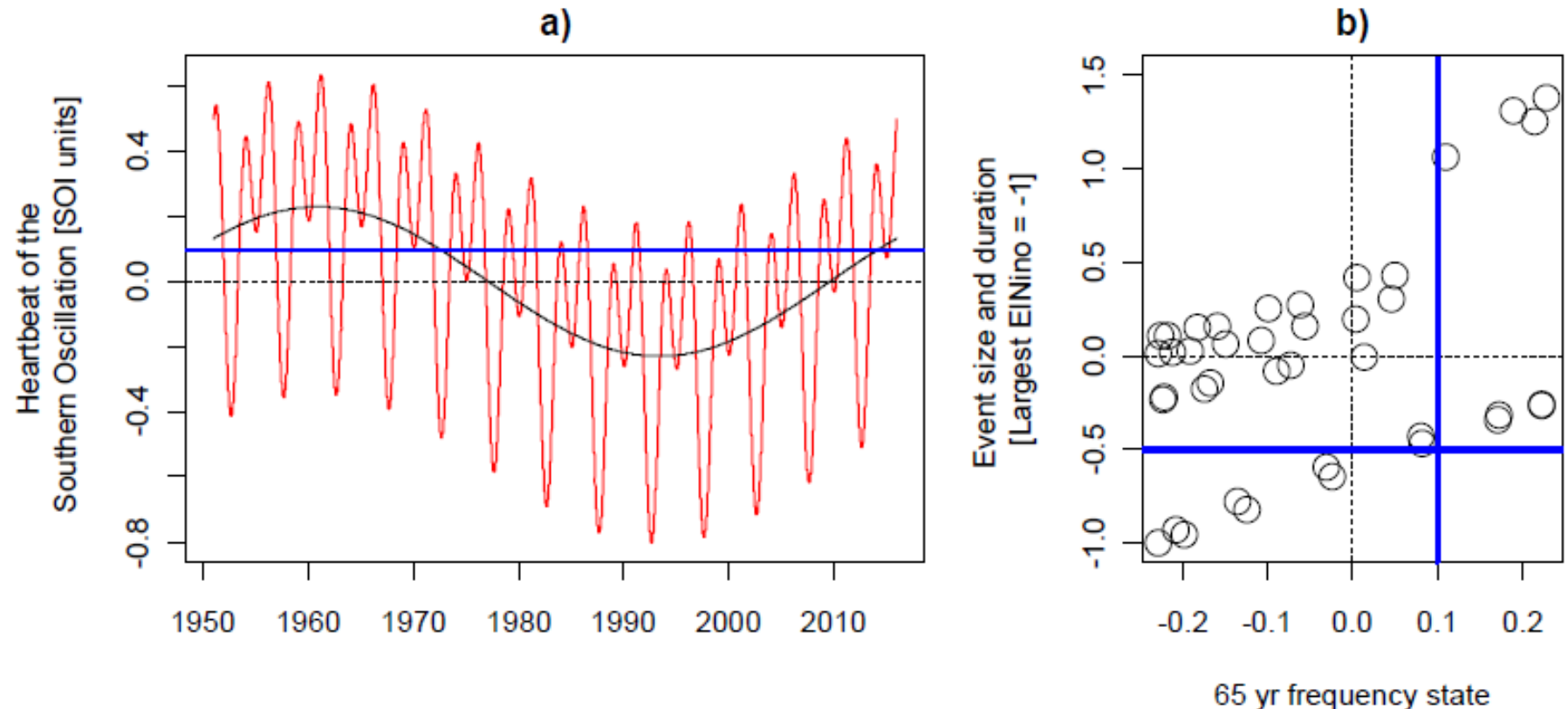
Multiple extensions: Temperature  $T(\mathbf{r}, t)$ , Velocity  $v(\mathbf{r}, t)$ , Biomes  $B(\mathbf{r}, t)$

**Outcome:** Greatly improved climate prediction and resolution from ensembles.

# The Heartbeat of the Southern Oscillation

The tropical ENSO attractor with 2.5, 5 and 65 year frequency states:

This attractor structure explains prolonged and contracted ENSO event modulation effects large:small El Niño 2:1 (La Niña 12:1) and the 1970's 'change of state' is a nonlinear threshold effect.

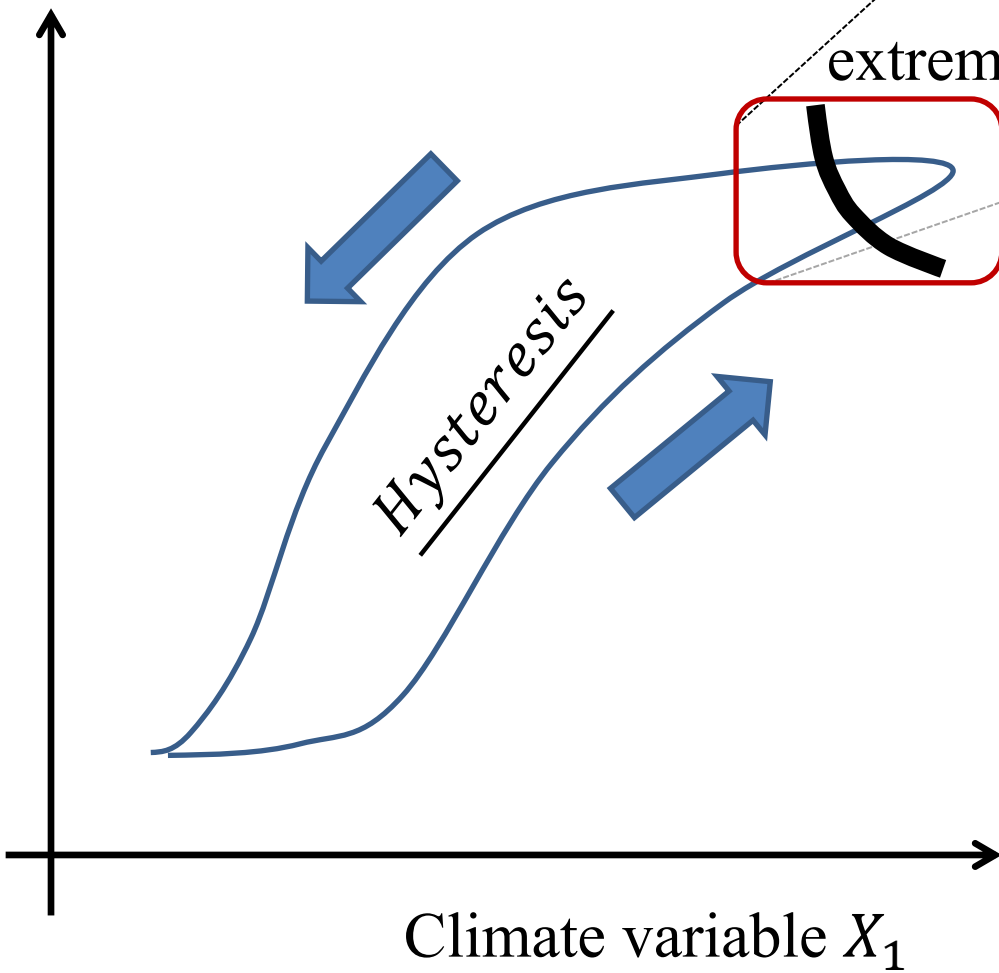


Current conditions indicate the ENSO process is in an unstable warm phase...



The ENSO climate hysteresis may be ‘tearing’  
The extreme range may change: implications for Sahel

Here is a visual explanation of extremes...  
Climate variable  $X_2$



*DFSA helps us interpret  
climate dynamics  $\mathbf{X}$   
and the extreme rainfall  $\mathbf{Z}$   
risk consequences*

$$\mathbf{X} \rightarrow \mathbf{Z}$$

$$\Pr(Z > z) = \int_{Z>z} f_X(\mathbf{x}, t) d\mathbf{x}dt$$

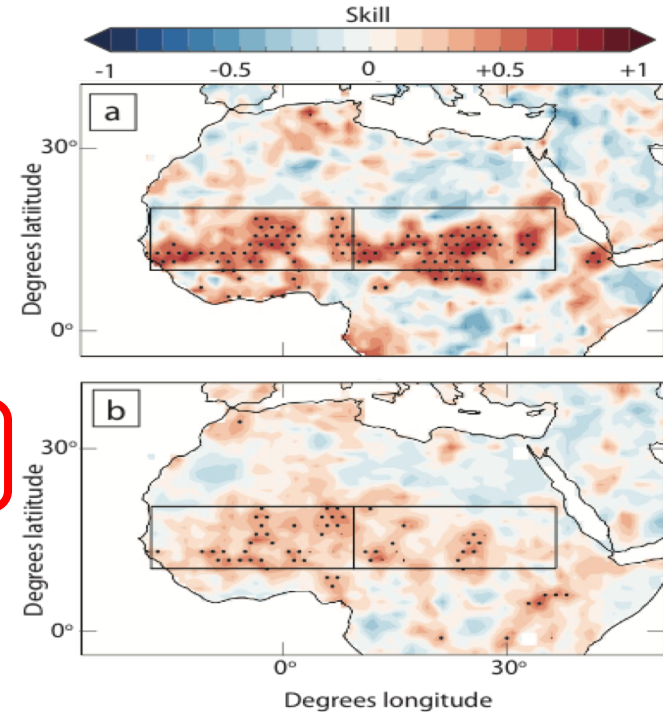
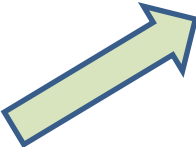
Bruun and Tawn (1998)

# Sahel rain budget, downscaling, DFSA and extremes

Sahel downscaling rain budget cyclic plus other decomposition maps:

$$Z(\mathbf{r}, t) = C_S(\mathbf{r}, t) + C_L(\mathbf{r}, t) + Trend(\mathbf{r}, t) + \eta(\mathbf{r}, t)$$

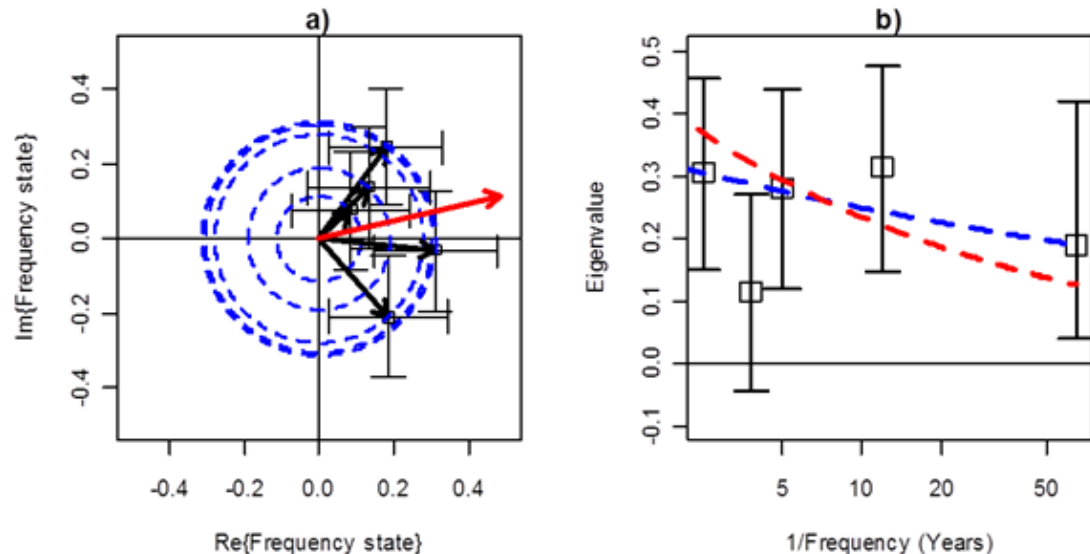
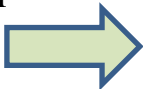
*Seasonal**Long term*



**Sahel skill assessment** (stipples show significant improvement in boxes above). Regional downscaling has shown potential for good predictive skill in these regions. Sheen *et al.* (2017)

## Sahel DFSA

Investigation looks at seasonal, long term and trend decomposition  
It is shown here for ENSO  
Bruun *et al.*, (2017)



# Food security and resilience:

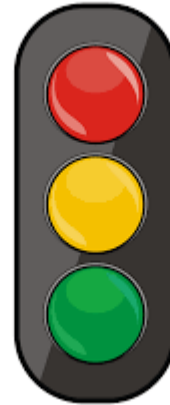
**Assisting Climate Adaptation with Communities In the African Sahel (ACACIAS)** (NERC/DIFD SHEAR proposal (PI: Sheen), prepared with M2D help, waiting result).



## ‘Traffic light’ risk charts of rain budget

Seasonal, long term modes (DFSAs)  
1/10 and 1/100 year event extreme  
risk level (rainfall and arid).

1954 (wet) vs 1984 (dry) assessed  
*Spatial DFSA enhances analysis capacity*



**Food security:** In Sahel countries Gross Domestic Product (GDP) is sensitive to agricultural production.

e.g. in Senegal a 20% change in agricultural growth  
contributes to a 3.5% change in GDP.

A 3.5 bn\$ per year is vulnerable to agricultural instability  
and hence climate volatility in Senegal and Ghana (our focus).

Wider analysis: 30bn\$ is at stake in whole Sahel  
these methods help better understand *resilience* for us all.

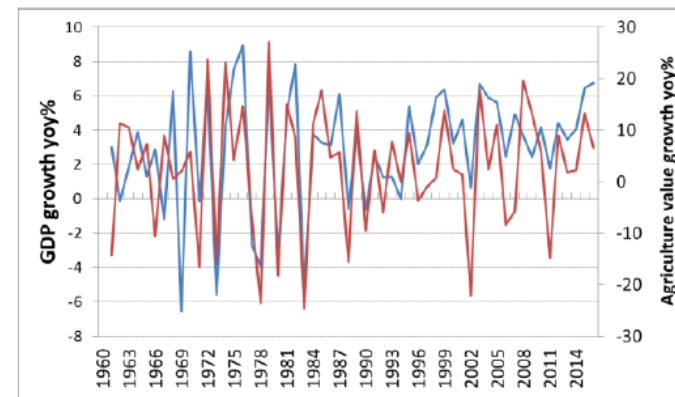


Figure 1: Senegal annual GDP growth (blue) and agricultural growth (red). Note high volatility during 1970-1980s drought period. Variability and correlation are quantified in Table 1. Data from World Bank, 2018.

# Summary

## Coffee cup science

## Fractals

- DFSA enables the accurate investigation of low amplitude and low frequency climatic modes
- It complements and extends current methods
- Multiple applications: ENSO, teleconnection, downscaling, ...
- Focus here: Resilience and food security to extreme climate changes
- Improved decision making with sophisticated yet simple models

