Effect of electrical resistivity on the damping of slow sausage modes

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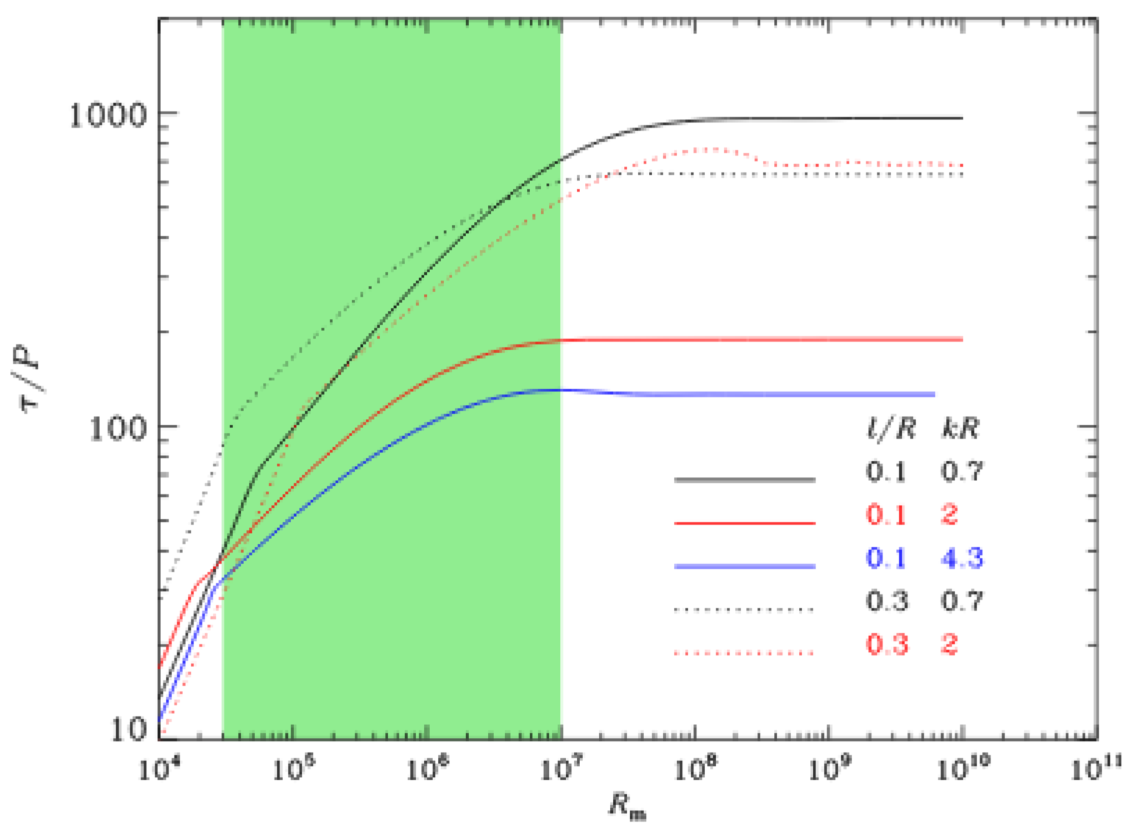
Motivation

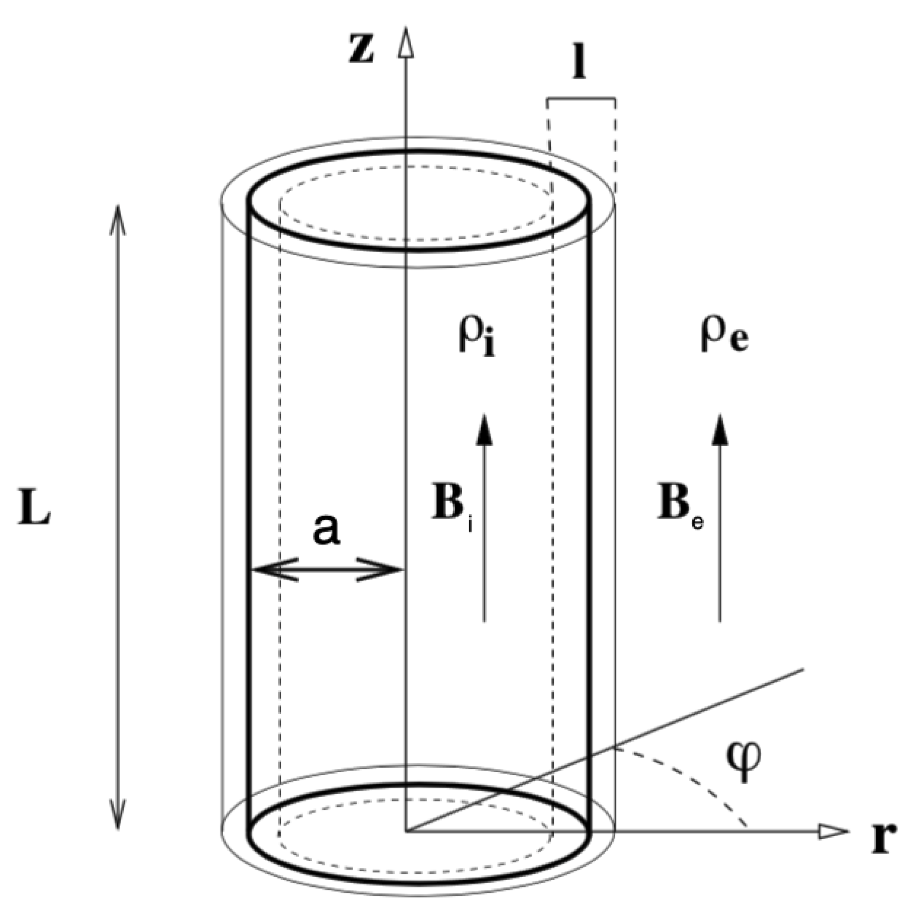
Dependence on **magnetic Reynolds number** of **damping-time-to-period ratio** of sausage modes in a solar photospheric pore studied numerically by [2] in the resistive ($\eta \neq 0$) MHD framework:



- Region of $R_m > 10^7$: damping almost independent of $\eta \Rightarrow$ mainly due to resonant absorption in cusp continuum
- Region of $R_m < 3 \cdot 10^4$: damping linearly dependent on $\eta \Rightarrow$ mainly due to resistive effects
- Green region of $3 \cdot 10^4 < R_m < 10^7$: intermediate regime where both electrical resistivity and resonant absorption are important for damping

Goal: to explain/confirm this behavior through an analytical model

Model



Straight cylinder aligned with equilibrium magnetic field, circular basis and discontinuous boundary ($l = 0$), as a model for the photospheric pore (figure modified from [1]). Inside (index "i") and outside (index "e") of cylinder are two distinct uniform plasmas.

Analytical derivations

- Every perturbed quantity is expressible in terms of $\nabla \cdot \xi = R(r) \exp\{i(k_z z - \omega t)\}$. The following solution is found for R:

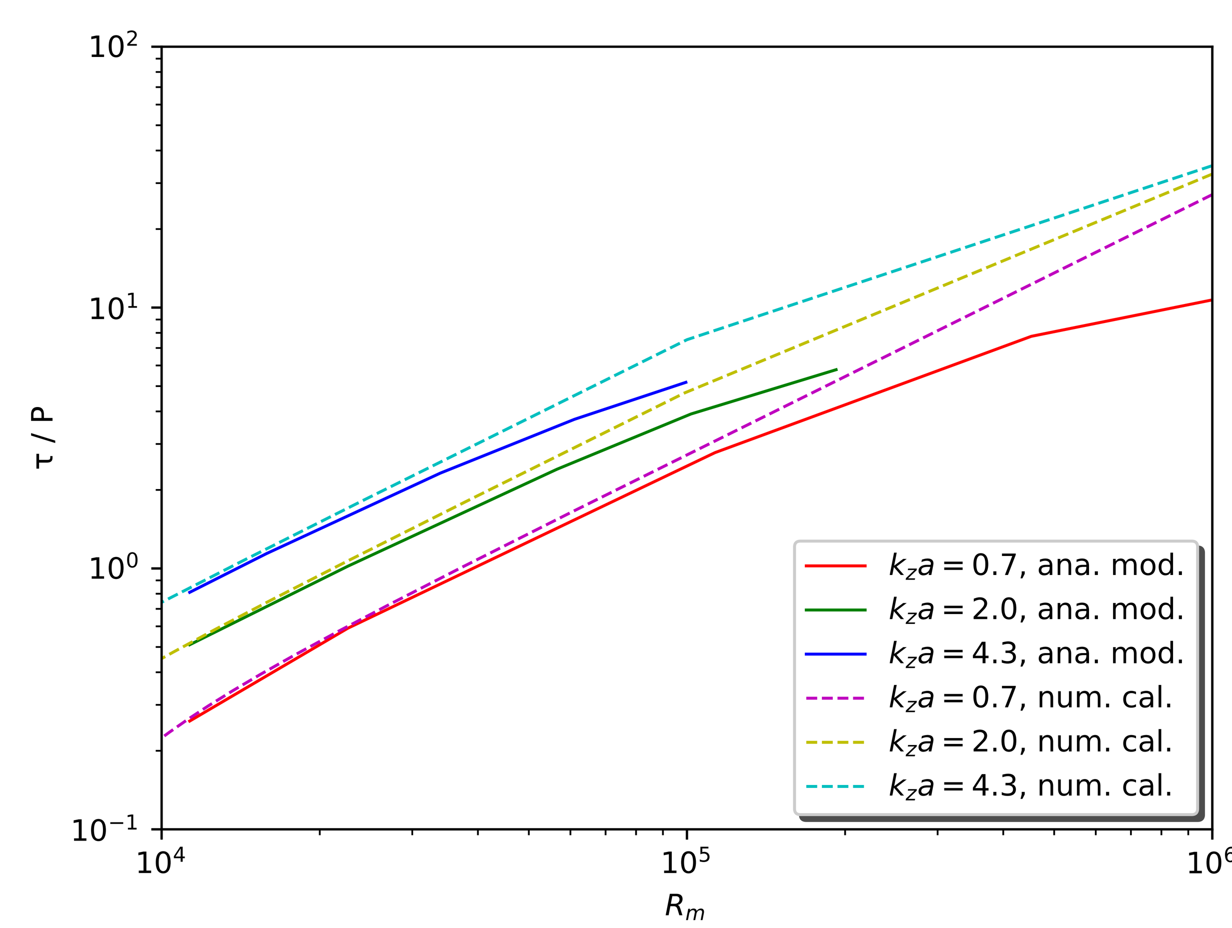
$$R(r) = \begin{cases} C_1\ J_0(\kappa_{-,\mathrm{i}} r) \ + \ C_3\ J_0(\kappa_{+,\mathrm{i}} r) & \text{if } r < a \\ C_2\ H_0(\kappa_{-,\mathrm{e}} r) \ + \ C_4\ H_0(\kappa_{+,\mathrm{e}} r) & \text{if } r > a \end{cases},$$

  where J_0 is Bessel function of first kind and order 0, and H_0 means either $H_0^{(1)}$ or $H_0^{(2)}$ (Hankel functions of order 0), depending on the sign of the imaginary part of its argument.
- κ_- represents the wave part of the solution, which is complex because of damping from resistivity. It can be thought of as the radial wavenumber

  κ_+ represents Hartmann layer (electromagnetic boundary layer with strong gradients which gets thinner as resistivity η becomes smaller)
- With proper boundary conditions, complex dispersion relation for ω can be derived
- Long wavelength limit formula of dispersion relation also available

Results

- Solving dispersion relation numerically allows to compare **damping-time-to period ratio** (τ/P) as a function of **magnetic Reynolds number** (R_m) from analytical model ("ana. mod.") of this research with numerical calculations ("num. cal.") from [2] code adapted to infinitely small transition layer ($l \approx 0$):



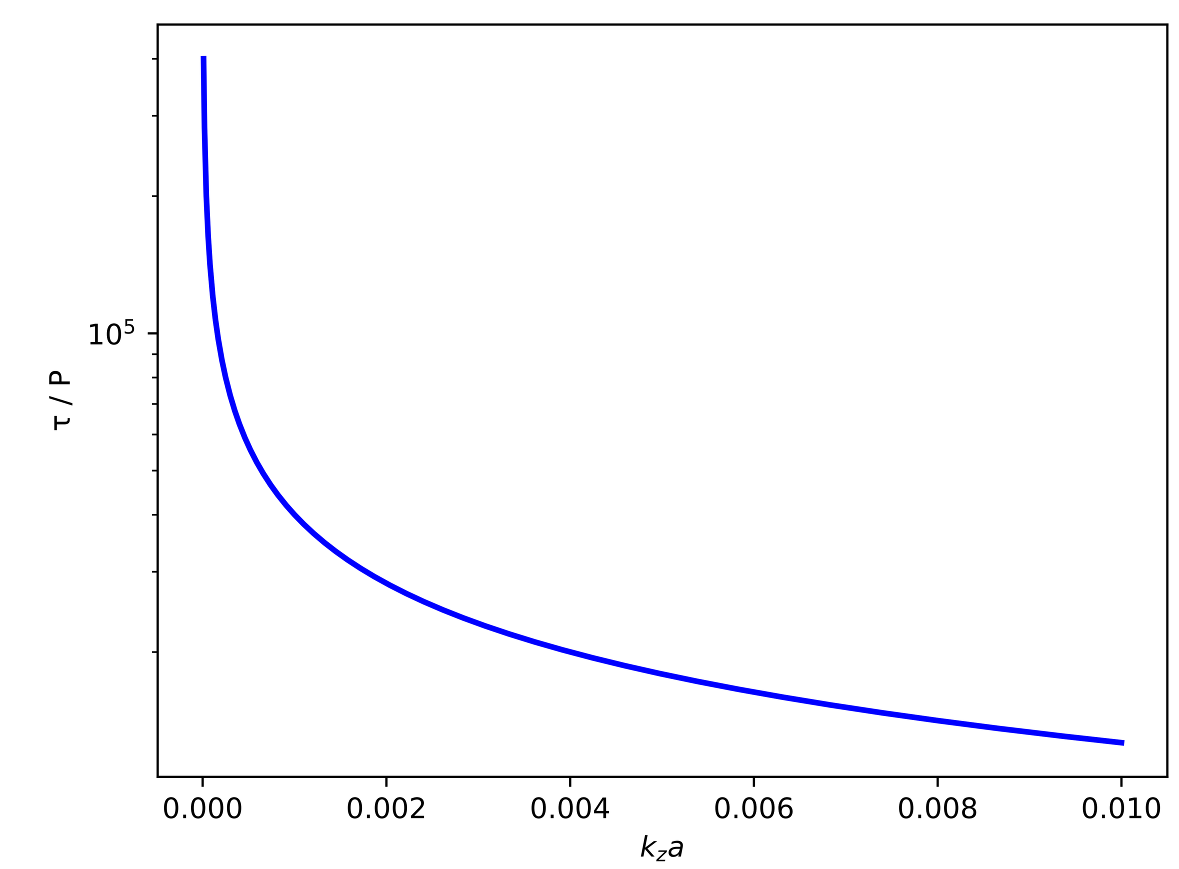
- In the long wavelength limit ($k_z a \to 0$), analytical results produce simpler formula for damping rate. Since ω close to internal cusp frequency (ω_{Ci}) in this limit, writing

$$\omega^2 = \omega_{\mathrm{Ci}}^2 (1 + \delta)$$

  we find:

$$\delta \propto \sqrt{\eta}$$

- Following figure is graph of **damping time-to-period ratio** in function of $\boldsymbol{k_z a}$ for long wavelength limit (in logarithmic scale):



In the long wavelength limit, the resistive damping thus vanishes.

References & acknowledgements

[1] Arregui, I. et al. "Resonantly damped fast MHD kink modes in longitudinally stratified tubes with thick non-uniform transitional layers". In: *Astronomy & Astrophysics* 441.1 (2005), pp. 361–370.

[2] Chen, S. et al. "Damping of Slow Surface Sausage Modes in Photospheric Waveguides". In: *The Astrophysical Journal* 868.1 (2018), p. 5.

Acknowledgements. We thank Marcel Goossens for discussions on wave modes in resistive MHD. This research was supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (TVD via grant agreement No 724326), which MG gratefully acknowledges for his phd studentship. This research was also supported by the C1 grant TRACEspace of Internal Funds KU Leuven (TVD), and by the National Natural Science Foundation of China (SXC: 41604145, BL:11761141002, 41674172).