A New Method for Obtaining Dispersion Diagrams for Vertical Magnetic Flux Tubes with Arbitrary Parameters

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INTRODUCTION

Magnetohydrodynamic (MHD) waves are considered to play an important role in coronal heating. Understanding the properties of these waves under a realistic model is vital in accurately determining their contribution to the energy budget of the solar atmosphere.

A generalised model for wave propagation in solar structures, as of yet, is not available. Realistic models are required such that they can be combined with observations allowing magnetoseismology to be conducted and provide more accurate information about the local plasma properties.

Dispersion diagrams display the properties of permittable trapped waves within a specific system. These diagrams are traditionally produced by obtaining the solutions to a transcendental dispersion equation relating wavenumber and frequency.

We introduce a new numerical method to obtain solutions for the sausage and kink modes on dispersion diagrams under arbitrary plasma structuring. This technique implements the shooting method to match necessary boundary conditions on displacement and total pressure of the waveguide. The proposed technique does not require an analytical dispersion relation – making it a powerful tool in the context of solar MHD wave analysis.

METHOD

Consider a 2D magnetic slab in planar coordinates such that the internal plasma properties are different to the external plasma properties. However, the plasma in both regions IS allowed to be inhomogeneous. Traditionally, a uniform plasma is modelled to simplify the mathematics involved.

Linearising the system of MHD equations results in a differential equation for the perturbation of horizontal velocity $\hat{v}_x$ (or displacement). The momentum equation also provides an expression for the total pressure perturbation $\hat{P}_T$. These two conditions are required to be satisfied in traditional methods of finding eigenmodes of the system. We implement these conditions by assuring that both expressions match at the boundaries of the slab.

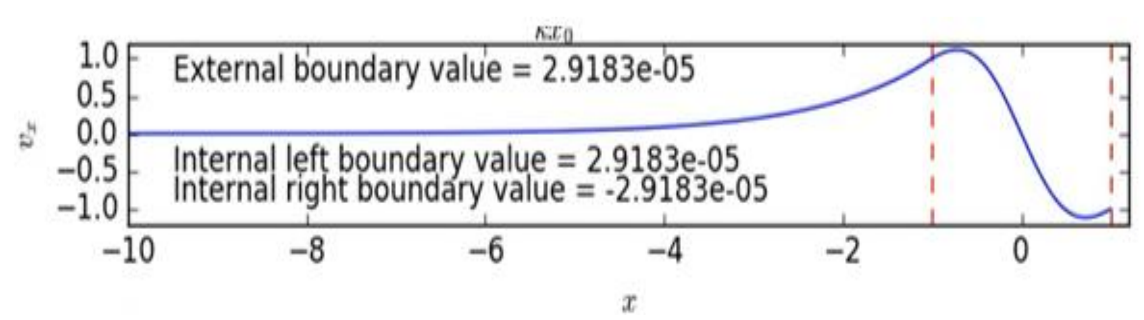

Figure 1: Example of numerical solution of $\hat{v}_x$ for the sausage mode. The sausage mode is the antisymmetric solution such that the velocity at one boundary is the negative of that at the opposite boundary.

Another important physical condition to account for is that wave amplitude decays infinitely far away from the waveguide boundaries, restricting solutions to exist inside the waveguide. Using this physical constraint we apply the shooting method from numerical infinity and solve the differential equation for the external region up to the boundary. Using the obtained values for $\hat{v}_x$ and $\hat{P}_T$ at the boundary, we "shoot" to the opposite boundary depending on whether the kink mode or sausage mode is being investigated.

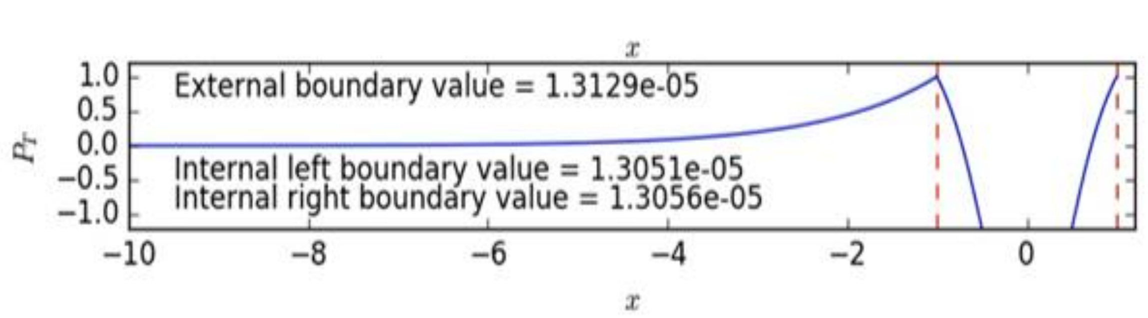

Figure 2: Example of numerical solution of $\hat{P}_T$ for the sausage mode.

VALIDATION

To check this proposed approach works it would be sensible to compare the solutions obtained with known results from previous analytical studies which make advantage of a dispersion relation. We consider the work on a uniform magnetic slab undertaken by Edwin & Roberts (1982) **[1]** . This work has provided the foundations for wave studies in the solar atmosphere. We apply the method and compare the solutions to the dispersion diagram obtained analytically.

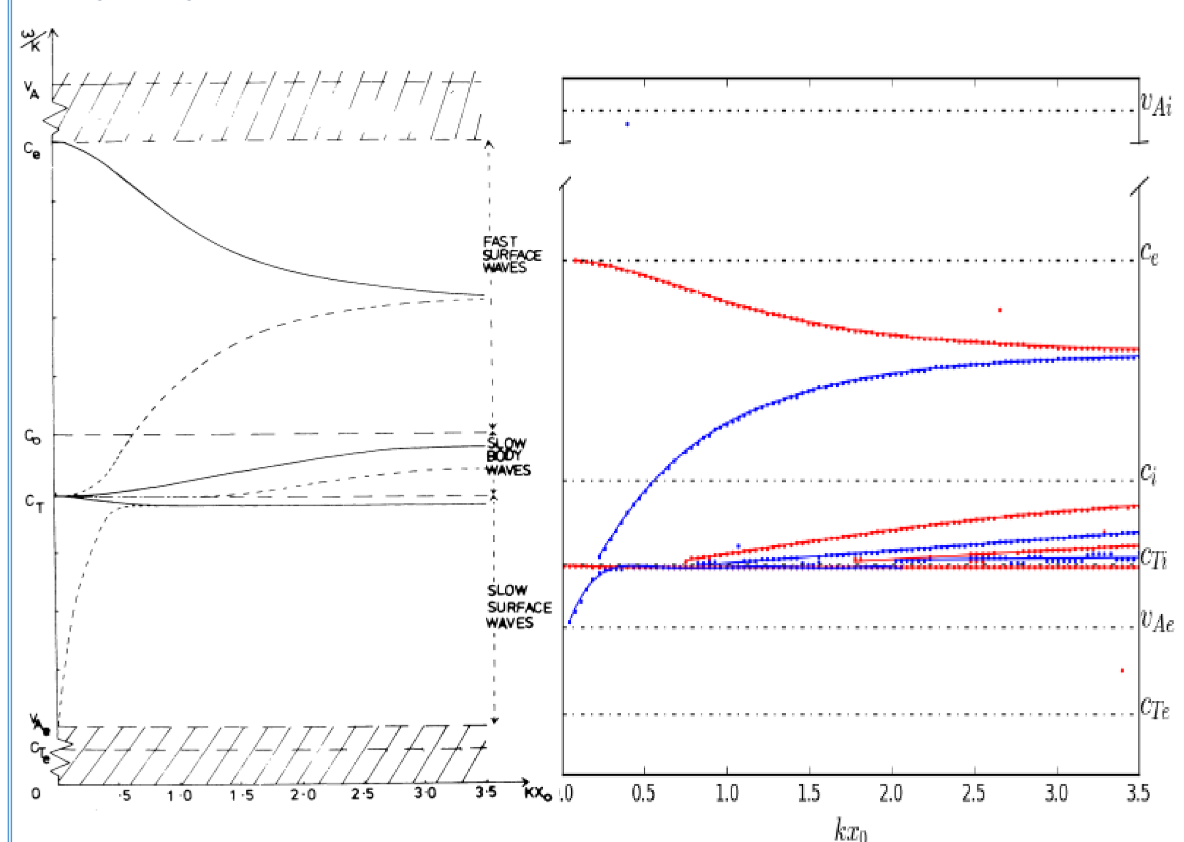

Figure 3: Dispersion diagram for magnetoacoustic waves under photospheric conditions. Phase speed solutions of dispersion relation (left), solid lines correspond to sausage mode, dash lines correspond to kink mode. Compared with solutions obtained by numerical method (right), red dots show sausage mode, blue dots show kink mode.

Figure 3 shows the solutions obtained by the proposed method compared to known analytical solutions under photospheric conditions for a uniform magnetic slab. The striking resemblance between the two diagrams and the ability to obtain the small band of body modes, which usually require a large numerical resolution, indicates the power of this technique.

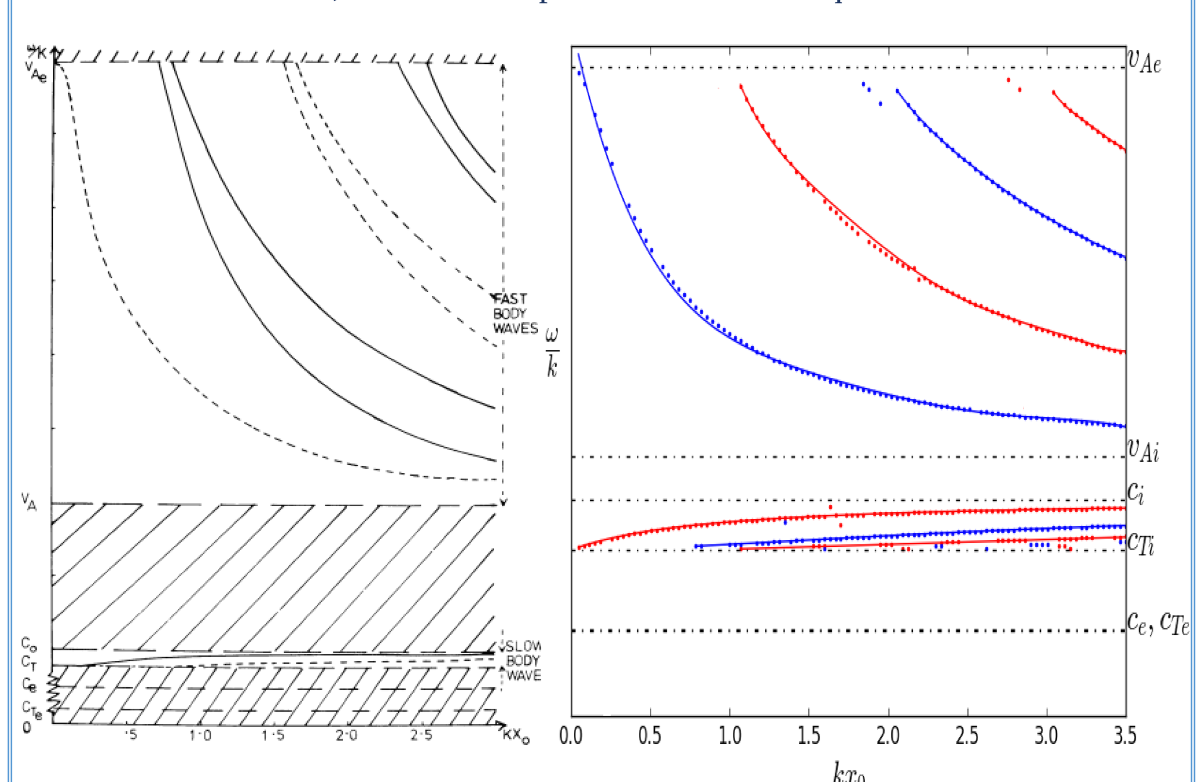

Figure 4: Same as Figure 3 but under coronal conditions.

Figure 4 shows the comparison under coronal conditions for the uniform magnetic slab.

The same procedures have been applied to a uniform slab in the presence of a steady background flow under both photospheric and coronal conditions as investigated in **[2]**. The results can be seen on supplementary material and again in both cases the solutions are accurately determined.

RESULTS

We model a magnetic slab in coronal conditions with an inhomogeneous internal density structuring. The external environment is kept uniform. The density is modelled as a Gaussian with a varying standard deviation. Large standard deviation corresponds to the uniform plasma case.

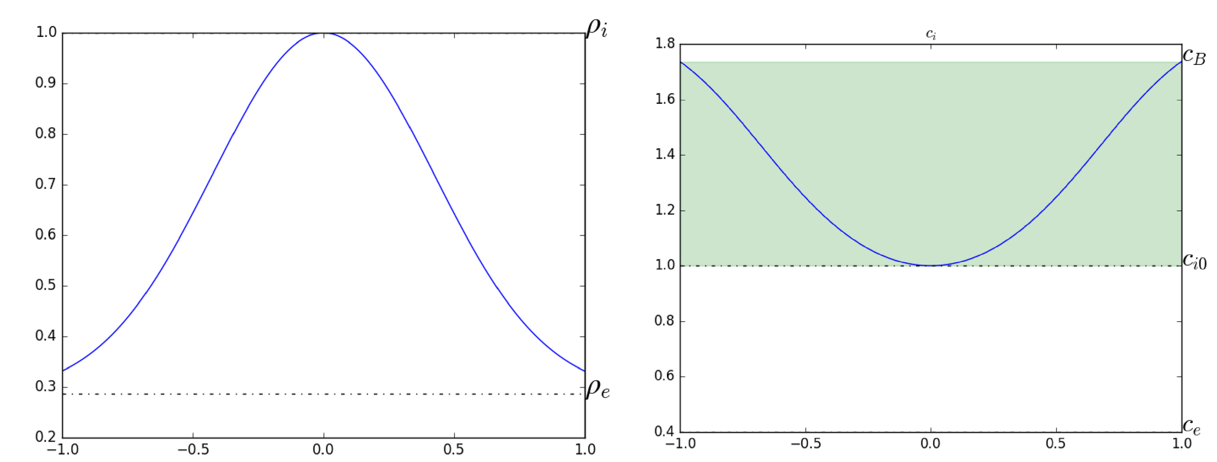

Figure 5: Left: Density profile for a coronal slab with a standard deviation of W = 0.6. Right: Corresponding internal sound speed profile for the given density structuring. Alfvén and Tube speeds show similar behaviour.

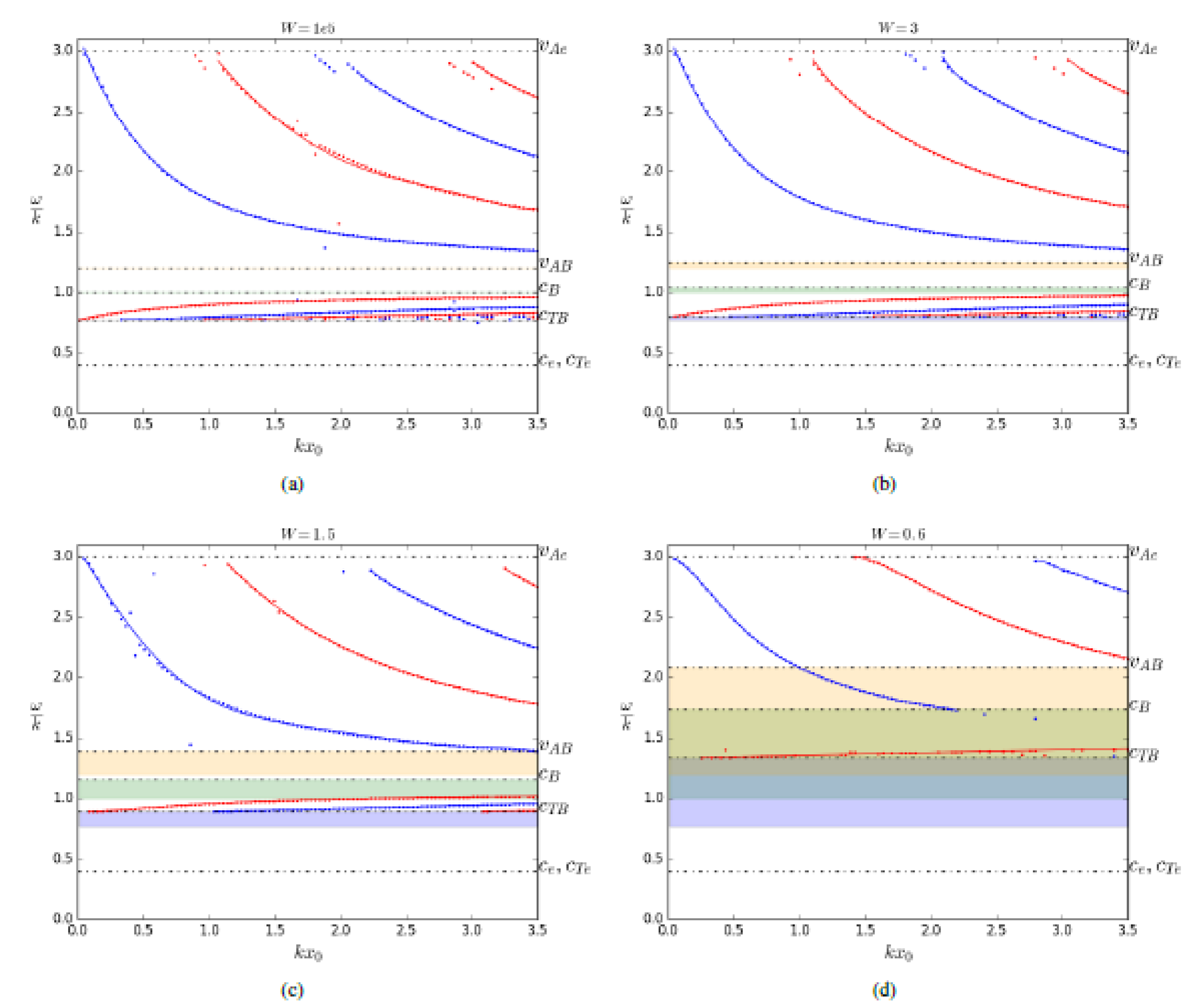

Figure 6: Dispersion Diagrams for different density Gaussians. (a) W = 1e5 (b) W =3 (c) W = 1.5 (d) W = 0.6. Shaded regions show area of inhomogeneity from boundary to max/min internal value for each speed. Red dots/curves = Sausage mode. Blue dots/curves = Kink mode.

In Figure 6 the resulting Dispersion Diagrams for a coronal slab with a Gaussian density profile are shown. The slow body waves remain trapped between the minimum internal sound speed (given by the maximum density value) and the tube speed at the boundary. Therefore as inhomogeneity is increased, this region becomes narrower. The fast body waves remain unaffected by the inhomogeneity until large standard deviations where they are cut-off by the sound speed at the boundary of the waveguide.

Figure 7 shows the same results under photospheric conditions. As expected, slow surface waves are cut-off by the tube speed at the boundary, such that these waves are no longer trapped under large inhomogeneities in density. Fast surface waves appear unaffected by the inhomogeneity. More analysis should be undertaken to accurately determine the behavior and physical properties of body modes under these conditions, as certain conditions allow the trapped nature of these modes.

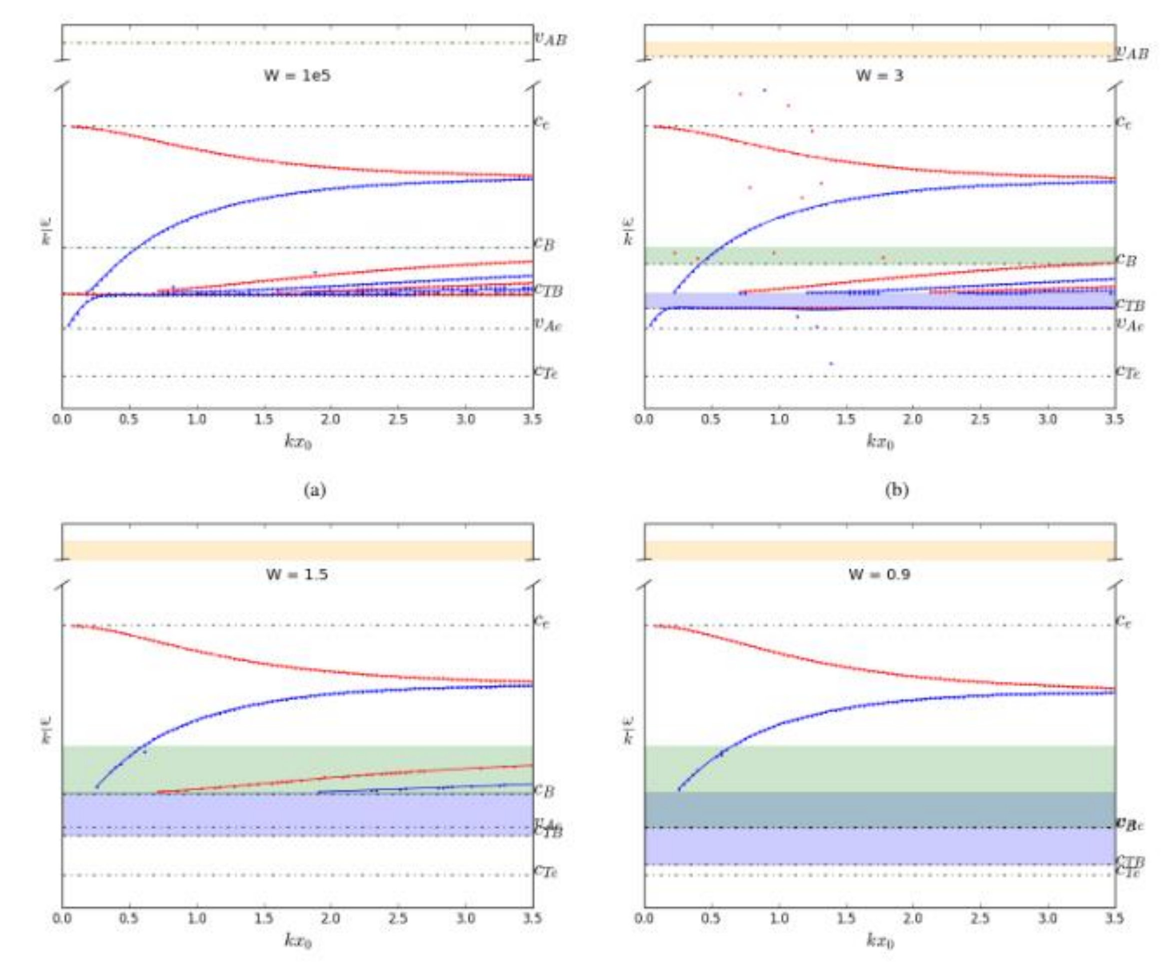

Figure 7: Dispersion Diagrams for different density Gaussians. (a) W = 1e5 (b) W =3 (c) W = 1.5 (d) W = 0.6. Shaded regions show area of inhomogeneity from boundary to max/min internal value for each speed. Red dots/curves = Sausage mode. Blue dots/curves = Kink mode.

CONCLUSIONS

- We propose a new numerical method to obtain the solutions on a dispersion diagram for any arbitrary model of a solar structure without the requirement of deriving the dispersion relation.
- The method accurately reproduces the solutions of known dispersion relations of a uniform magnetic slab including under the presence of a bulk background flow for both photospheric and coronal conditions.
- We implement the technique to investigate trapped wave modes of an inhomogeneous model with a Gaussian distribution of internal density. We find that as the standard deviation (width) of the inhomogeneity is decreased there are physical effects to the permittable wave modes.
- Under coronal conditions, slow body waves are trapped between the minimum value of internal sound speed where the density is at a maximum and the boundary value of the internal tube speed. Fast body waves experience a cut off at the boundary value of internal sound speed.
- Under photospheric conditions, slow surface waves are cut off by the internal tube speed at the boundary. Fast surface waves appear unaffected by the inhomogeneity. More investigation should be undertaken to examine the properties of body modes under these conditions.

REFERENCES

[1] Edwin, P. M. & Roberts, B. 1982, Sol. Phys., 76, 239

[2] Nakariakov, V. M. & Roberts, B. 1995a, Sol. Phys., 159, 213

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