

Vortex Detection by improved Gamma Method

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1. Introduction

The Vortex Detection Code (VDC) aims to detect and identify the vortex flow motions in the 2D numerical observational solar data. Inspired by the Γ method (Graftieaux et al. 2001) and the convolution Γ_1 (Fernando Zigynov. 2019), we improve the gamma function and combine with new design vortex boundary algorithm to tracking more accurate detection result in observational data.

4. Detection result

The observation data comes from CRISP. The image scale of the CRISP observations is 0.059 per pixel and the mean cadence is about 8.25s. VDC will track most of the vortex that it detects and record each of their lifetime, change of area, gamma 1 value, boundary, radius velocity and tangential velocity during its lifetime.

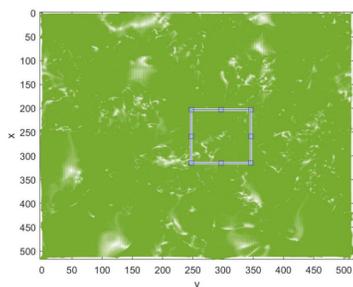


Figure 2: The region of interest (ROI) is marked by rectangle.

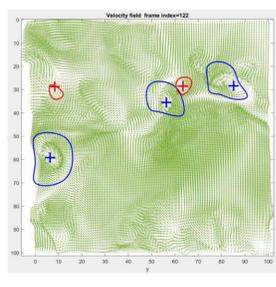


Figure 3: The snapshot of the detected vortices boundaries and centers. Red color corresponds to the clockwise rotated vortex and blue to the counter-clockwise rotation.

2. Advanced Γ Method

2.1 Original Γ method:

Γ_1 is a dimensionless scalar function about point P , where S is a two-dimensional area surrounding P . M lies in S , PM denote the displacement vector from point P to a point M in S . UM denotes the velocity vectors at M ; Z is a unit vector normal to the plane. A is the area of S ; Up is the local convection velocity around the vortex center

$$\Gamma_1(P) = \frac{1}{A} \int_{M \in S} \frac{(PM \times UM) \cdot z}{\|PM\| \|UM\|} dS = \frac{1}{A} \int_S \sin(\theta_M) dS$$

$$\Gamma_2(P) = \frac{1}{A} \int_{M \in S} \frac{(PM \times (UM - Up)) \cdot z}{\|PM\| \|UM - Up\|} dS$$

2.2 Convolution Γ method:

Consider only longitudinal and latitude velocity (2-D velocity field). The Γ method can be expressed in 2-D with convolution as follows. Computation with convolution will speed up the computation by 3 orders at least than just using the original discrete form of the function.

$$\Gamma_1(P) = \frac{1}{N} \frac{PM_x \otimes U_{Mx} - PM_y \otimes U_{My}}{\|PM\| \otimes \|UM\|}$$

$$\Gamma_2(P) = \frac{1}{N} \frac{PM_x \otimes U_{Mx} - U_{Mx} \otimes PM_x - PM_y \otimes U_{My} + U_{My} \otimes PM_y}{\|PM\| \otimes \|UM - U_p\|}$$

U_{Mx} longitudinal velocity field component
 U_{My} latitude velocity field component
 PM_x x component of radius vectors from center P to all potential M point, current point P is the origin
 PM_y y component of radius vectors from center P to all potential M point, current point P is the origin
 $\|\cdot\|$ magnitude
 N the number of points in the current velocity fields that around the vortex center P .

\otimes denotes the convolution operator

2.3 Advanced Γ method:

We notice that convolution Γ method using two convolution kernel $\frac{PM_x}{\|PM\|}$ and $\frac{PM_y}{\|PM\|}$. According to the convolution theorem, we also notice that kernel are separable if the rank of it is 1. But we notice that only the numerator part could be separable. In order to use the computation advantage of the separable convolution. We improve the algorithm by estimate a reasonable single scalar Z to approximate the denominator in the kernel. As a result we got the advanced form of it.

w_1 and w_2 as two 1-D vector

$$PM_x = w_1 \times w_2$$

$$PM_y = (w_1 \times w_2)'$$

$$\Gamma_1(P) = \frac{1}{Z \sqrt{N}} \left[(w_1 \otimes \frac{U_{Mx}}{\|UM\|} \otimes w_2) - (w_2 \otimes \frac{U_{My}}{\|UM\|} \otimes w_1) \right]$$

$$\Gamma_2(P) = \frac{1}{Z \sqrt{N}} \left[(w_1 \otimes \frac{U_{Mx}}{\|UM\|} \otimes w_2) - (w_2 \otimes \frac{U_{My}}{\|UM\|} \otimes w_1) \right]$$

Z estimates scalar rate that approximate displacement magnitude matrix $\|PM\|$, which enable Γ_{1adv} and Γ_{2adv} bound by 1.

By using the separable convolution kernel, it will be faster at least $m/2$ times than the convolution form, where m is the size of the 1-D vector.

3. Unfixed Circular Area

Traditionally, the Γ method utilizes the same/similar fixed-size S as in the velocity field that sampling from the PIV procedure. Adopting the same fixed-size S does not influence on finding the vortex center in the Γ_1 method. However, in Γ_2 , it involves the local convection velocity around the center point. Traditional Γ_2 will take all points in area S into account for calculating the local convection velocity UP , even the point local in the bottom right corner of area S . Thus, the spatial geometric relationship between suspect point M and other points in area S will influence the result of UP since it is traditionally defined as the local convection velocity field around vortex center P .

Thus we introduce the unfixed circular area in Γ_2 when calculating gamma 2. VDC will self-design the area that in gamma 2 and thus provide a better boundary detection result. Figure 1 shows one of the examples when detecting the boundary, where the blue point indicates the circular area, this area will vary according to the radius from the current suspect boundary point to the vortex center.

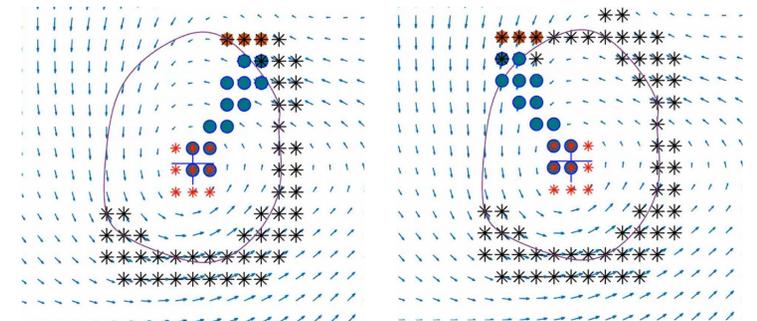


Figure 1. Unfixed Circular Area.

Blue point indicates the unfixed circular section that is used to compute the local convection velocity of the current boundary point. Black point indicates the searching process.

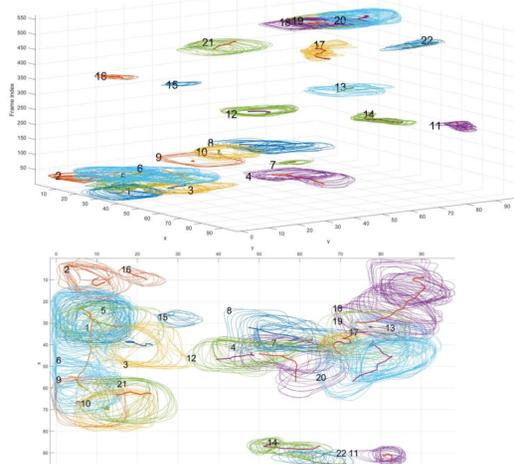


Figure 4: The region of interest (ROI). The top panel (a) and bottom panel (b) show all the detected vortices in ROI. The red lines correspond to the trajectory of the vortices centers. The vertical axis of the left panel indicates the number of frames in observational data, where the mean cadence is about 8.25s.

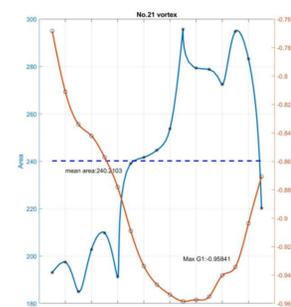
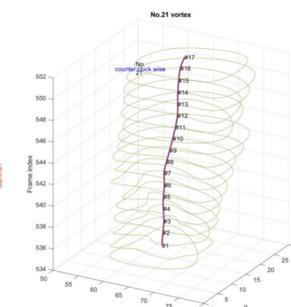
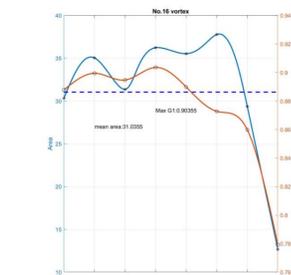
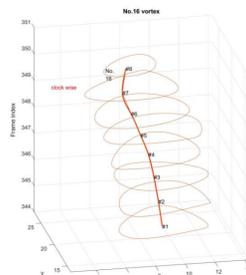


Figure 5: Vortex boundary change as a function of time. The two isolated vortices No. 16 and No. 21 are shown on the left and right panels respectively.

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